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S-matrix interpretation of finite-temperature real-time field theories

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Abstract

The aim of the article is to construct the S-matrix interpretation of the perturbation theory for the Wigner functions generating functional at a finite temperature. The temperature is introduced in the theory by the way typical for the microcanonical description. The perturbation theory contains the two-temperature Green functions. The two possible boundary conditions are considered. One of them is usual in a field theory vacuum boundary condition. The corresponding generating functional can be used in the particle physics. Another type of the boundary condition assumes that the system under consideration is in environment of the free particles background field. This leads to the theory with Kubo-Martin-Schwinger boundary conditions ones at the one-temperature limit.

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1 Introduction

The field-theoretical description of statistical systems at a finite temperature is usually based on the formal analogy between imaginary time and inverse temperature β ($\beta = 1/T$) [1]. This approach is fruitful [2] for description of the static properties of a system, but it demands a complicated mathematical apparatus for the analytic continuation to the real time [3], if we want to clear up the dynamical aspects. At the same time a great number of modern problems is connected with system dynamics. It is important, for instance, for description of evolution of the Universe, of quark-gluon plasma behavior, etc.

The first important quantitative attempt to build the real-time finite-temperature field theory [4] discover a problem of the pinch-singularities. The further investigation of the theory have allowed to demonstrate the cancellation mechanism of these unphysical singularities [5]. This attained by doubling of the degrees of freedom [6, 7]: the Green functions of the theory represent 2×2 matrix. It surely makes the theory more complicated, but the operator formalism of the thermo-field dynamics [8] shows the unavoidable character of this complication.

It is important to note that the traditional real-time finite-temperature field-theoretical description [6, 7] of statistical systems based on the Kubo-Martin-Schwinger (KMS) [6, 9, 10] condition for a field:

$$\Phi(t) = \Phi(t - i\beta) \tag{1.1}$$

which, without fail, leads to the *equilibrium* fluctuation-dissipation conditions [11] (see also [12]).

We shell use the S-matrix approach which is natural for description of time evolution. (The S-matrix description was used also in [13, 14].) For this purpose the amplitudes

$$< (p)_m|(q)_n >= a_{n,m}(p_1, p_2, ..., p_m; q_1, q_2, ..., q_n) \tag{1.2}$$

of the n- into m-particles transition will be introduced (Sec.2). The in- and out-states must be composed from mass-shell particles [15]. Using these amplitudes we will calculate the probability which is

$$\sim |a_{n,m}|^2 =< (p)_m|(q)_n >< (q)_n|(p)_m > . \tag{1.3}$$

This will lead to the doubling of degrees of freedom.

The finite temperature will be introduced (see also [16]) taking into account that, for instance,

$$d\Gamma_n = |a_{n,m}|^2 \prod_1^n \frac{d^3q_i}{(2\pi)^3 2\epsilon(q_i)}, \quad \epsilon(q) = (q^2 + m^2)^{1/2}, \tag{1.4}$$

is the differential measure of the initial state. Then we will define the temperature as the function of initial energy through the equation of state (see Sec.2). This introduction of temperatures as the Lagrange multiplier is obvious for microcanonical description [9].

The final-state temperature will be introduced by the same way. So, we will construct the two-temperature theory. In this theory with two temperatures (the equation of state

can be applied at the very end of calculations) it is impossible to use the KMS boundary condition.

One can note that the product of amplitudes (1.3) describes the closed-path motion in the functional space, with some "turning-point" fields $\Phi(\sigma_\infty)$, where σ_∞ is the infinitely far hypersurface (Sec.3). The value of $\Phi(\sigma_\infty)$ specify the evironment of a system.

In Sec.2 the vacuum boundary condition $\Phi(\sigma_\infty) = 0$, familiar for a field theory, will be considered. This theory can be applied in the particle physics. The simplest (minimal) choice of $\Phi(\sigma_\infty) \neq 0$ assumes that the system under consideration is surrounded by black-body radiation (Sec.4). This interpretation restores Niemi-Semenoff's formulation of the theory [5].

One should admit also that this choice of boundary condition is not unique: one can consider another organization of the environment of considered system. The S-matrix interpretation will be used since it able to show the way of adoption of the formalism to the arbitrary boundary conditions. It should broaden the potentialities of the real-time finite-temperature field-theoretical methods. The special interest represent the topological effects, but in this paper consideration will be performed in the perturbation framework only.

2 Vacuum boundary condition

The starting point of our calculations is n- into m-particles transition amplitude $a_{n,m}$, the derivation of which is well known procedure in the perturbation framework. Let us introduce $(n+m)$-point Green function $G_{n,m}$ through a generating functional Z_j [17]:

$$G_{n,m}((x)_n;(y)_m) = (-i)^{n+m} \prod_{k=1}^{n} \hat{j}(x_k) \prod_{k=1}^{m} \hat{j}(y_k) Z_j, \tag{2.1}$$

where

$$\hat{j}(x) = \frac{\delta}{\delta j(x)}, \tag{2.2}$$

and

$$Z_j = \int D\Phi e^{iS_j(\Phi)}. \tag{2.3}$$

The action

$$S_j(\Phi) = S(\Phi) - V(\Phi) + \int dx j(x)\Phi(x), \tag{2.4}$$

where $S(\Phi)$ is the free part and $V(\Phi)$ describes the interactions. At the end one can put $j = 0$.

To provide the convergence of the integral (2.3) over scalar field Φ the action $S_j(\Phi)$ must contain positive imaginary part. Usually for this purpose Feynman's $i\varepsilon$-prescription is used. It is better for us to shift infinitesimally the time contour to the upper half plane [18, 3], i.e. on the following contour $C_+ : t \to t + i\epsilon, \quad \epsilon > 0$ and after all calculations to return the time contour on the real axis, $\epsilon \to +0$.

In eq. (2.3) the integration is performed over all field configurations with standard vacuum boundary condition:

$$\int d^4x\partial_\mu(\Phi\partial^\mu\Phi) = \int_{\sigma_\infty} d\sigma_\mu\Phi\partial^\mu\Phi = 0, \tag{2.5}$$

which prosides to zero contribution from the surface term.

Let us introduce now field ϕ through the equation:

$$-\frac{\delta S(\phi)}{\delta\phi(x)} = j(x) \tag{2.6}$$

and perform the shift $\Phi \to \Phi + \phi$ in integral (2.3), conserving boundary condition (2.5). Considering ϕ as the probe field created by the source:

$$\begin{aligned} \phi(x) &= \int dy G_0(x-y)j(y), \\ (\partial^2 + m^2)_x G_0(x-y) &= \delta(x-y), \end{aligned} \tag{2.7}$$

the only connected Green function $G^c_{n,m}$ will be interesting for us. Therefore,

$$G^c_{n,m}((x)_n;(y)_m) = (-i)^{n+m}\prod_{k=1}^{n}\hat{j}(x_k)\prod_{k=1}^{m}\hat{j}(y_k)Z(\phi), \tag{2.8}$$

where

$$Z(\phi) = \int D\Phi e^{iS(\Phi)-iV(\Phi+\phi)} \tag{2.9}$$

is the new generating functional.

To calculate the nontrivial elements of S-matrix we must put the external particles on the mass shell. Formally this procedure means amputation of the external legs of $G^c_{n,m}$ and further multiplication on the free particles wave functions. In result the amplitude of n- into m-particles transition $a_{n,m}$ in the momentum representation has the form:

$$a_{n,m}((q)_n;(p)_m) = (-i)^{n+m}\prod_{k=1}^{n}\hat{\phi}(q_k)\prod_{k=1}^{m}\hat{\phi}^*(p_k)Z(\phi). \tag{2.10}$$

Here we introduce the operator

$$\hat{\phi}(q) = \int dx e^{-iqx}\hat{\phi}(x), \quad \hat{\phi}(x) = \frac{\delta}{\delta\phi(x)} \tag{2.11}$$

and q_k and p_k are the momentum of in- and out-going particles.

Supposing that the momentum of particles are insufficient for us the probability of n- into m-particles transition is defined by the integral:

$$r_{n,m} = \frac{1}{n!m!}\int d\omega_n(q)d\omega_m(p)\delta^{(4)}(\sum_{k=1}^{n}q_k - \sum_{k=1}^{m}p_k)|a_{n,m}|^2, \tag{2.12}$$

where

$$d\omega_n(q) = \prod_{k=1}^{n} d\omega(q_k) = \prod_{k=1}^{n} \frac{d^3 q_k}{(2\pi)^3 2\epsilon(q_k)}, \quad \epsilon = (q^2 + m^2)^{1/2}, \tag{2.13}$$

is the Lorentz-invariant phase space element. We assume that the energy-momentum conservation δ-function was extracted from the amplitude.

Note that $r_{n,m}$ is the divergent quantity. To avoid this problem with trivial divergence let us divide the energy-momentum fixing δ-function into two parts:

$$\delta^{(4)}(\sum q_k - \sum p_k) = \int d^4 P \delta^{(4)}(P - \sum q_k)\delta^{(4)}(P - \sum p_k) \tag{2.14}$$

and consider a new quantity:

$$r(P) = \sum_{n,m} \frac{1}{n!m!} \int d\omega_n(q) d\omega_m(p) \delta^{(4)}(P - \sum_{k=1}^{n} q_k)\delta^{(4)}(P - \sum_{k=1}^{n} p_k)|a_{n,m}|^2. \tag{2.15}$$

Here we suppose that the number of particles are not fixed. It is not too hard to see that, up to phase space volume,

$$r = \int d^4 P \; r(P) \tag{2.16}$$

is the imaginary part of amplitude $< vac|vac >$. Therefore, computing $r(P)$ the standard renormalization procedure can be applied and the new divergences will not arise in our formalism.

The Fourier transformation of δ-functions in (2.15) allows to write $r(P)$ in the form:

$$r(P) = \int \frac{d^4\alpha_1}{(2\pi)^4} \frac{d^4\alpha_2}{(2\pi)^4} e^{iP(\alpha_1+\alpha_2)} R(\alpha_1, \alpha_2), \tag{2.17}$$

where

$$R(\alpha_1, \alpha_2) = \sum_{n,m} \frac{1}{n!m!} \int \prod_{k=1}^{n} \{d\omega(q_k) e^{-i\alpha_1 q_k}\} \prod_{k=1}^{m} \{d\omega(p_k) e^{-i\alpha_2 p_k}\} |a_{n,m}|^2. \tag{2.18}$$

The introduction of the "Fourier-transformed" probability $R(\alpha_1, \alpha_2)$ means only that the phase-space volume is not fixed exactly, i.e. it is proposed that 4-vector P is fixed with some accuracy if α_i are fixed. The energy and momentum in our approach are still locally conserved quantities since an amplitude a_{nm} is translational invariant. So, we can perform the transformation:

$$\alpha_1 \sum q_k = (\alpha_1 - \sigma_1) \sum q_k + \sigma_1 \sum q_k \to (\alpha_1 - \sigma_1) \sum q_k + \sigma_1 P \tag{2.19}$$

since 4-momenta are conserved. The choice of σ_1 fixes the reference frame. This degree of freedom of the theory was considered in [19, 20].

Inserting (2.10) into (2.18) we find that

$$R(\alpha_1, \alpha_2) = \exp\{i \int dx dx' (\hat{\phi}_+(x) D_{+-}(x - x', \alpha_2) \hat{\phi}_-(x') - \\ \hat{\phi}_-(x) D_{-+}(x - x', \alpha_1) \hat{\phi}_+(x'))\} Z(\phi_+) Z^*(\phi_-), \tag{2.20}$$

where D_{+-} and D_{-+} are the positive and negative frequency correlation functions correspondingly:

$$D_{+-}(x-x',\alpha) = -i\int d\omega(q)e^{iq(x-x'-\alpha)} \tag{2.21}$$

describes the process of particles creation at the time moment x_0 and its absorption at x'_0, $x_0 > x'_0$, and α is the center of mass (CM) 4-coordinate. Function

$$D_{-+}(x-x',\alpha) = i\int d\omega(q)e^{-iq(x-x'+\alpha)} \tag{2.22}$$

describes the opposite process, $x_0 < x'_0$. These functions obey the homogeneous equations:

$$(\partial^2 + m^2)_x G_{+-} = (\partial^2 + m^2)_x G_{-+} = 0 \tag{2.23}$$

since the propagation of mass-shell particles is described.

We suppose that $Z(\phi)$ may be computed perturbatively. For this purpose the following transformation will be used:

$$\begin{aligned} e^{-iV(\phi)} = e^{-i\int dx\hat{j}(x)\hat{\phi}'(x)}e^{i\int dxj(x)\phi(x)}e^{-iV(\phi')} = \\ e^{\int dx\phi(x)\hat{\phi}'(x)}e^{-iV(\phi')} = \\ e^{-iV(-i\hat{j})}e^{i\int dxj(x)\phi(x)}, \end{aligned} \tag{2.24}$$

where $\hat{j}$ was defined in (2.2) and $\hat{\phi}$ in (2.11). At the end of calculations the auxiliary variables j, ϕ' can be taken equal to zero. Using the first equality in (2.24) we find that

$$Z(\phi) = e^{-i\int dx\hat{j}(x)\hat{\Phi}(x)}e^{-iV(\Phi+\phi)}e^{-\frac{i}{2}\int dxdx'j(x)D_{++}(x-x')j(x')}, \tag{2.25}$$

where D_{++} is the causal Green function:

$$(\partial^2 + m^2)_x G_{++}(x-y) = \delta(x-y) \tag{2.26}$$

Inserting (2.25) into (2.20) after simple manipulations with differential operators, see (2.24) we find the expression:

$$\begin{aligned} R(\alpha_1,\alpha_2) = e^{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)}\times \\ \times\exp\{\frac{i}{2}\int dxdx'(j_+(x)D_{+-}(x-x',\alpha_1)j_-(x') - j_-(x)D_{-+}(x-x',\alpha_2)j_+(x') - \\ -j_+(x)D_{++}(x-x')j_+(x') + j_-(x)D_{--}(x-x')j_-(x'))\}, \end{aligned} \tag{2.27}$$

where

$$D_{--} = (D_{++})^* \tag{2.28}$$

is the anticausal Green function. One can consider $R(\alpha_1,\alpha_2) = R(\alpha_1,\alpha_2;j_1,j_2)$ as the generating functional for Wigner functions (see also [3, 21]).

Considering the system with large number of particles we can simplify calculations choosing the CM frame $P = (P_0 = E, \vec{0})$. It is useful also [16, 9] to rotate the contours

of integration over $\alpha_{0,k}$: $\alpha_{0,k} = -i\beta_k, Im\beta_k = 0, k = 1, 2$. In result, omitting unnecessary constant, we will consider $R = R(\beta_1, \beta_2)$.

External particles play the double role in the S-matrix approach: their interactions create and annihilate the system under consideration and, on the other hand, they are probes through which the measurement of a system is performed. Since β_k are the conjugate to the particles energies quantities we will interpret them as the inverse temperatures in the initial (β_1) and final (β_2) states of interacting fields. But there is the question: are constants β_k really the "good" parameters to describe the system.

The integrals over β_k:

$$r(E) = \int \frac{d\beta_1}{2\pi i}\frac{d\beta_2}{2\pi i} e^{(\beta_1+\beta_2)E} e^{-F(\beta_1,\beta_2)}, \tag{2.29}$$

where

$$F(\beta_1, \beta_2) = -\ln R(\beta_1, \beta_2), \tag{2.30}$$

can be computed by the stationary phase method. This assumes that the total energy E is a fixed quantity. The solutions of the equations (of state):

$$E = \frac{\partial F(\beta_1, \beta_2)}{\partial \beta_k}, \quad k = 1, 2, \tag{2.31}$$

gives the mostly probable values of β_k at a given E. Eqs. (2.31) always have the real solutions and, because of energy conservation law, both eqs. (2.31) have the same solution with the property [9]:

$$\beta_k = \beta(E), \quad \beta > 0. \tag{2.32}$$

If the fluctuations of β_k are large it is insufficient to know $\beta(E)$: the expansion of integral (2.29) over $(\beta - \beta_k)$ will lead to the asymptotic series with zero convergence radius since $F(\beta_1, \beta_2)$ is essentially nonlinear function. In this paper we assume that β is the "good" parameter, i.e. the fluctuations of β_k are Gaussian. In this case we can interpret $F(\beta_1, \beta_2)$ as the free energy and $1/\beta_k$ as the temperatures. Such definition of thermodynamical parameters is in a spirit of microcanonical description. Note the important role of decomposition (2.14) in this interpretation of the β_k.

The structure of generating functional (2.27) is the same as the generating functional of Niemi-Semenoff [5] have. The difference is only in the definition of Green functions which follows from the choice of boundary condition (2.5). The Green functions $D_{ij}, i, j = +, -$ were defined on the time contours $C_\pm$ in the complex time plane ($C_- = C_+^*$). This definition of the time contours coincide with Keldysh' time contour [7]. The expression (2.27) can be written in the compact form if the matrix notations are used. Note also a doubling of the degrees of freedom. This doubling is unavoidable since Green functions D_{ij} are singular on the light cone.

3 Closed-path boundary conditions

The calculation of generating functional $R(\alpha_1, \alpha_2)$ is performed introducing the corresponding generating functional

$$R_0(\phi_\pm) = Z(\phi_+)Z^*(\phi_-) =$$
$$\int D\Phi_+ D\Phi_- \exp\{iS(\Phi_+) - iS(\Phi_-) - iV(\Phi_+ + \phi_+) + iV(\Phi_- + \phi_-)\}, \tag{3.1}$$

see (2.20). The fields ϕ_+, ϕ_- and Φ_+, Φ_- were defined on the time contours C_+, C_-. By definition, path integral (3.1) describes the closed path motion in the space of fields Φ. We want to use this fact and introduce a more general boundary condition which also guaranties the cancelation of the surface terms in the perturbation framework. We will introduce the equality:

$$\int_{\sigma_\infty} d\sigma_\mu \Phi_+ \partial^\mu \Phi_+ = \int_{\sigma_\infty} d\sigma_\mu \Phi_- \partial^\mu \Phi_-. \tag{3.2}$$

The solution of eq.(3.2) requires that the fields Φ_+ and Φ_- (and theirs first derivatives $\partial_\mu \Phi_\pm$) coincide on the boundary hypersurface σ_∞:

$$\Phi_\pm(\sigma_\infty) = \Phi(\sigma_\infty), \tag{3.3}$$

where, by definition, $\Phi(\sigma_\infty)$ is the arbitrary, "turning-point", field.

In absence of the surface terms, the existence of nontrivial field $\Phi(\sigma_\infty)$ has the influence only on the structure of Green functions

$$G_{++} =< T\Phi_+\Phi_+ >, \quad G_{+-} =< \Phi_+\Phi_- >,$$
$$G_{-+} =< \Phi_-\Phi_+ >, \quad G_{--} =< \tilde{T}\Phi_-\Phi_- > \tag{3.4}$$

where $\tilde{T}$ is the untitemporal time ordering operator. This Green functions must obey the equations:

$$(\partial^2 + m^2)_x G_{+-}(x-y) = (\partial^2 + m^2)_x G_{-+}(x-y) = 0,$$
$$(\partial^2 + m^2)_x G_{++}(x-y) = (\partial^2 + m^2)^*_x G_{--}(x-y) = \delta(x-y), \tag{3.5}$$

and the general solution of these equations:

$$G_{ii} = D_{ii} + g_{ii},$$
$$G_{ij} = g_{ij}, \quad i \neq j \tag{3.6}$$

contain the undefined terms g_{ij} which are the solutions of homogenous equations:

$$(\partial^2 + m^2)_x g_{ij}(x-y) = 0, \quad i,j = +,-. \tag{3.7}$$

The general solution of these equations (they are distinguished by the choice of the time contours $C_\pm$)

$$g_{ij}(x-x') = \int d\omega(q) e^{iq(x-x')} n_{ij}(q) \tag{3.8}$$

are defined through the functions n_{ij} which are the functionals of "turning-point" field $\Phi(\sigma_\infty)$: if $\Phi(\sigma_\infty) = 0$ we must have $n_{ij} = 0$ and we will come back to the theory of previous section.

Our aim is to define n_{ij}. We can suppose that $n_{ij} \sim< \Phi(\sigma_\infty)\cdots\Phi(\sigma_\infty) >$. The simplest supposition gives:

$$n_{ij} \sim< \Phi_i\Phi_j > \sim< \Phi^2(\sigma_\infty) > . \tag{3.9}$$

We will find the exact definition of n_{ij} starting from the S-matrix interpretation of the theory.

4 KMS boundary condition

In the previous section it was shown that the theory permits the arbitrariness of boundary condition: the turning-point field $\Phi(\sigma_\infty)$ may be arbitrary since the "closed-path" motion in the functional space is described. We will suppose that on the infinitely far hypersurface σ_∞ there are only free, mass-shell, particles. Formally it follows from (3.6) -(3.8). This assumption is natural also in the S-matrix framework [15]. In other respects the choice of boundary condition is arbitrary.

Therefore, our aim is concerned with description of evolution of the system in a background field of mass-shell particles. In this paper we will assume that there are not any special correlations among background particles. We will take into account only the restrictions connected with energy-momentum conservation laws. Quantitatively this means that multiplicity distribution of background particles is Poison-like, i.e. is determined by the mean multiplicity only. This is in spirit of definition of n_{ij} in eqs. (3.8), (3.9).

Our derivation is the same as in [20]. Here we restrict ourselves mentioning only the main quantitative points.

Calculating the product $a_{n,m}a^*_{n,m}$ we describe a process of particles creation and further their adsorption. In the vacuum case of Sec.2 the two process were taken into account: creation of particles by the $a_{n,m}$ and theirs adsorption by $a^*_{n,m}$, and the opposite process of particles creation by $a^*_{n,m}$ and theirs adsorption by $a_{n,m}$. This processes were time ordered. This was the reason of the frequency correlation functions D_{+-} and D_{-+} appearance. In the nonvacuum case, i.e. in presence of the background particles, this time-ordered picture is slurring over since the possibility to absorb particles before their creation appears.

The processes of creation and adsorption are described in vacuum by the product of operator exponents of $\hat\phi_+\hat\phi_-$, $\hat\phi_-\hat\phi_+$. We can derive (see also [20]) the generalizations of (2.20): presence of the background particles will lead to the following generating functional:

$$R_{cp} = e^{iN(\hat\phi_i\hat\phi_j)}R_0(\phi_\pm), \tag{4.1}$$

where $R_0(\phi_\pm)$ is the generating functional for vacuum case, see (3.1). The operator $N(\hat\phi^*_i\hat\phi_j), i,j = +,-$, describes the external particles environment.

The operator $\hat\phi^*_i(q)$ can be considered as the creation and $\hat\phi_i(q)$ as the annihilation operator, see definition (2.10). Correspondingly the product $\hat\phi^*_i(q)\hat\phi_j(q)$ acts as the activity

operator. So, in the expansion of $N(\hat{\phi}_i^*\hat{\phi}_j)$ we can leave only the first nontrivial term:

$$N(\hat{\phi}_i^*\hat{\phi}_j) = \int d\omega(q)\hat{\phi}_i^*(q)n_{ij}\hat{\phi}_j(q), \tag{4.2}$$

since no special correlation among background particles should be expected. If the external (nondynamical) correlations are present then the higher powers of $\hat{\phi}_i^*\hat{\phi}_j$ will appear in expansion (4.2). Following to the interpretation of $\hat{\phi}_i^*\hat{\phi}_j$ we conclude that n_{ij} is the mean multiplicity of background particles.

In (4.2) the normalization condition:

$$N(0) = 0 \tag{4.3}$$

was used and summation over all i, j was assumed. In the vacuum case only the combinations $i \neq j$ are present since the time ordering.

Computing R_{cp} we must conserve the translational invariance of amplitudes. It lead to extraction of the energy-momentum conservation δ-functions. Having background particles flow it is important to note that only connected contributions must be taken into account. Using the Fourier transformation we can find that to each vertex of in-going in $a_{n,m}$ particle we must adjust the factor $e^{-i\alpha_1 q/2}$ and for each out-going particle we have correspondingly $e^{-i\alpha_2 q/2}$.

So, the product $e^{-i\alpha_k q/2}e^{-i\alpha_j q/2}$ can be interpreted as the probability factor of the one-particle ($creation + annihilation$) process. The n-particles ($creation + annihilation$) process' probability is the simple product of these factors if there is not the special correlations among background particles. This interpretation is evident in the CM frame $\alpha_k = (-i\beta_k, \vec{0})$.

After this preliminaries it is not too hard to find that in the CM frame we have:

$$n_{++}(q_0) = n_{--}(q_0) = \frac{\sum_{n=0}^{\infty} n e^{-\frac{\beta_1+\beta_2}{2}|q_0|n}}{\sum_{n=0}^{\infty} e^{-\frac{\beta_1+\beta_2}{2}|q_0|n}} = \frac{1}{e^{\frac{\beta_1+\beta_2}{2}|q_0|} - 1} = \tilde{n}(|q_0|\frac{\beta_1+\beta_2}{2}). \tag{4.4}$$

Computing n_{ij} for $i \neq j$ we must take into account that we have one more particle:

$$n_{+-}(q_0) = \theta(q_0)\frac{\sum_{n=1}^{\infty} n e^{-\frac{\beta_1+\beta_1}{2}q_0 n}}{\sum_{n=1}^{\infty} e^{-\frac{\beta_1+\beta_1}{2}q_0 n}} + \Theta(-q_0)\frac{\sum_{n=0}^{\infty} n e^{\frac{\beta_1+\beta_1}{2}q_0 n}}{\sum_{n=0}^{\infty} e^{\frac{\beta_1+\beta_1}{2}q_0 n}} = \Theta(q_0)(1+\tilde{n}(q_0\beta_1)) + \Theta(-q_0)\tilde{n}(-q_0\beta_1) \tag{4.5}$$

and

$$n_{-+}(q_0) = \Theta(q_0)\tilde{n}(q_0\beta_2) + \Theta(-q_0)(1+\tilde{n}(-q_0\beta_2)). \tag{4.6}$$

Using (4.4), (4.5) and (4.6), and the definition (3.6) we find the Green functions:

$$G_{i,j}(x-x', (\beta)) = \int \frac{d^4q}{(2\pi)^4} e^{iq(x-x')}\tilde{G}_i j(q, (\beta)) \tag{4.7}$$

where

$$i\tilde{G}_i j(q,(\beta)) = \begin{pmatrix} \frac{i}{q^2-m^2+i\epsilon} & 0 \\ 0 & -\frac{i}{q^2-m^2-i\epsilon} \end{pmatrix} +$$

$$+2\pi\delta(q^2-m^2)\begin{pmatrix} \tilde{n}(\frac{\beta_1+\beta_2}{2}|q_0|) & \tilde{n}(\beta_2|q_0|)a_+(\beta_2) \\ \tilde{n}(\beta_1|q_0|)a_-(\beta_1) & \tilde{n}(\frac{\beta_1+\beta_2}{2}|q_0|) \end{pmatrix} \tag{4.8}$$

and

$$a_\pm(\beta) = -e^{\frac{\beta}{2}(|q_0|\pm q_0)}. \tag{4.9}$$

The corresponding generating functional has the standard form:

$$R_{cp}(j_\pm) = \exp\{-iV(-i\hat{j}_+) + iV(-i\hat{j}_-)\} \times$$
$$\times \exp\{\frac{i}{2}\int dxdx' j_i(x)G_{ij}(x-x',(\beta))j_j(x')\} \tag{4.10}$$

where the summation over repeated indexes is assumed.

Inserting (4.10) in the equation of state (2.31) we can find that $\beta_1 = \beta_2 = \beta(E)$. If $\beta(E)$ is a "good" parameter then $G_{ij}(x-x';\beta)$ coincide with the Green functions of the real-time finite-temperature field theory and the KMS boundary condition:

$$G_{+-}(t-t') = G_{-+}(t-t'-i\beta), \quad G_{-+}(t-t') = G_{+-}(t-t'+i\beta), \tag{4.11}$$

is restored. The eq.(4.11) can be deduced from (4.8) by the direct calculations.

5 Conclusion

In our interpretation of the real-time finite-temperature field theory the statistics and fields quantum dynamics were unlinked: statistics is fixed by the operator $\exp\{iN(\hat{\phi}_i^*\hat{\phi}_j\}$ and a pure field-theoretical dynamics is described by $R_0(\phi_\pm) = Z(\phi_+)Z^*(\phi_-)$, where $Z(\phi_\pm)$ is the vacuum into vacuum transition amplitude in presence of the external (auxiliary) fields $< vac|vac >_\phi$. We can say that the operator $\exp\{iN(\hat{\phi}_i\hat{\phi}_j)\}$ maps the system of interacting fields on the state with definite thermodinamical parameters.

This means the following procedure: we can act by the operator exponent $\exp\{N\}$ before calculation of R_0. We find in this case from (4.10) that

$$R_{cp}(\beta_1,\beta_2) = \int D\Phi_+ D\Phi_- e^{i\tilde{S}(\Phi_+)-i\tilde{S}(\Phi_-)}e^{i\Psi(\beta;\Phi_\pm)}, \tag{5.1}$$

where $\tilde{S}(\Phi)$ is the total action and

$$\Psi(\beta;\Phi_\pm) = -i\ln\{e^{iV(\Phi_++\phi_+)-iV(\Phi_-+\phi_-)}$$
$$e^{i\int dYdy[\hat{\phi}_i(Y+y/2)G_{ij}(y,(\beta))\hat{\phi}_j(Y-y/2)}e^{-iV(\Phi_++\phi_+)+iV(\Phi_-+\phi_-)}\} \tag{5.2}$$

is act as the effective source of fields $\Phi_\pm$ and depends from the temperature $1/\beta_i$. Such representation introduce in quantization procedure the external conditions of the considered problem. This can help to calculate the observables with high accuracy.

The formalism allows to describe an arbitrary system. The special interest presents the "local equilibrium" case. The structure of R_{cp} and of Green functions remains the same for this case, see (4.10) and (4.8), but β_k becomes coordinate dependent: $\beta_k = \beta_k(Y)$. Note that $Y = (x + x')/2$ is the Wigner's coordinate [14, 21]. The derivation of this result will be given in the following paper.

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